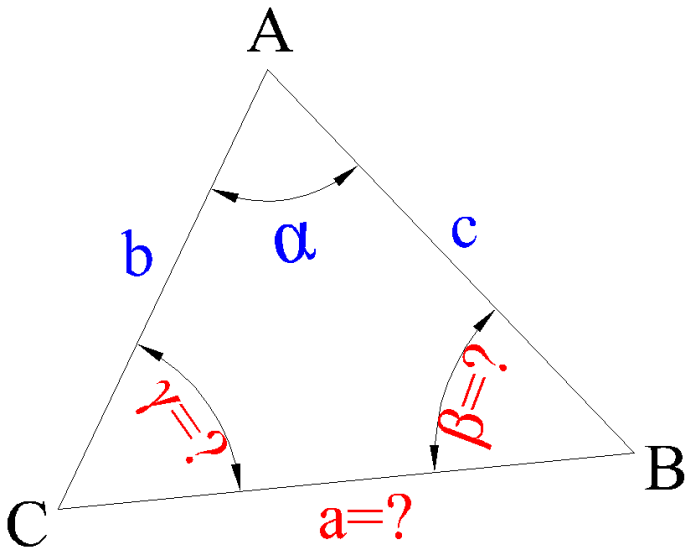
Rešavanje trougla u kome su izmerene dve strane i zahvaćeni ugao

-Tangensna teorema (14. Trigonometrijski obrazac)-



Poznato:

stana c
strana b
zahvaćeni ugao (α)

Nepoznato:

strana a
ugao β
ugao γ

Zadatak 1

Rešiti trougao primenom tangensne teoreme, ako su poznati sledeći podaci:

ugao α = 71°12'46", strana b=280,74m i strana c=247,66m

1) Unosimo poznate vrednosti u 14. trigonometrijski obrazac :

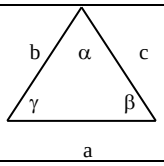
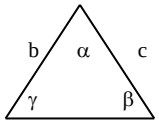
Trigonometrijski obrazac br.14							
Redni broj računanja i skica trougla	Strane i uglovi su uzeti:	b α c γ β a	$\frac{1}{2}(\beta+\gamma)=\frac{1}{2}\pi-\frac{1}{2}\alpha$ $\operatorname{tg}\frac{1}{2}(\beta-\gamma)=\frac{b-c}{b+c}\operatorname{ctg}\frac{1}{2}\alpha$		$\beta=\frac{1}{2}(\beta+\gamma)+\frac{1}{2}(\beta-\gamma)$ $\gamma=\frac{1}{2}(\beta+\gamma)-\frac{1}{2}(\beta-\gamma)$		$a=b\cdot\frac{\sin\alpha}{\sin\beta}$ $=c\cdot\frac{\sin\alpha}{\sin\gamma}$
b α c γ β a		½ α	° ' "	b	280,74	b	
				c	247,66	sinβ	
		½ (β+γ)		b-c		sinα	
		½ (β-γ)		b+c		sinγ	
		α	71°12'46"			c	
		β				a	
		γ		ctg½ α			
		π	180 00 00	tg½ (β-γ)		a	

2) Računamo razliku i zbir poznatih strana, kao i polovinu ugla α, zatim dobijene vrednosti unosimo u odgovarajuće polje obrasca.

b-c=280,74-247,66= +33,08 (znak se obavezno piše u obrascu, jer znak može biti negativan)

b+c=280,74+247,66= +528,40

½ α = $\frac{71^o12'46''}{2}$ = 35º36'23"

Trigonometrijski obrazac br.14							
Redni broj računanja i skica trougla	Strane i uglovi su uzeti:		$\frac{1}{2}(\beta+\gamma)=\frac{1}{2}\pi-\frac{1}{2}\alpha$ $\operatorname{tg}\frac{1}{2}(\beta-\gamma)=\frac{b-c}{b+c}\operatorname{ctg}\frac{1}{2}\alpha$		$\beta=\frac{1}{2}(\beta+\gamma)+\frac{1}{2}(\beta-\gamma)$ $\gamma=\frac{1}{2}(\beta+\gamma)-\frac{1}{2}(\beta-\gamma)$		$a=b\cdot\frac{\sin\alpha}{\sin\beta}$ $=c\cdot\frac{\sin\alpha}{\sin\gamma}$
			º ' "	b	280,74	b	
		½ α	35º36'23"	c	247,66	sinβ	
		½ (β+γ)		b-c	+33,08	sinα	
		½ (β-γ)		b+c	+528,40	sinγ	
		α	71º12'46"			c	
		β				a	
		γ		ctg½ α			
		π	180 00 00	tg½ (β-γ)		a	

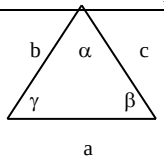
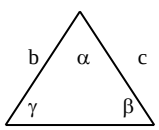
3) Računamo ctg½ α, $\frac{1}{2}(\beta+\gamma)$ i $\frac{1}{2}(\beta-\gamma)$:

ctg½ α= (tg(35º36'23"))$^{-1}$ =0,71610^{-1}= 1,39646

$$\frac{1}{2}(\beta+\gamma)=\frac{1}{2}\pi-\frac{1}{2}\alpha\rightarrow\frac{180^o}{2}-35^o36'23''\rightarrow\frac{1}{2}(\beta+\gamma)=54^o23'37''$$

$$\operatorname{tg}\frac{1}{2}(\beta-\gamma)=\frac{b-c}{b+c}\operatorname{ctg}\frac{1}{2}\alpha\rightarrow\frac{33,08}{528,40}\cdot1,39646\rightarrow\operatorname{tg}\frac{1}{2}(\beta-\gamma)=0,08742$$

$$\frac{1}{2}(\beta-\gamma)=\operatorname{arctg}(0,08742)=4^o59'47''$$

Trigonometrijski obrazac br.14							
Redni broj računanja i skica trougla	Strane i uglovi su uzeti:		$\frac{1}{2}(\beta+\gamma)=\frac{1}{2}\pi-\frac{1}{2}\alpha$ $\operatorname{tg}\frac{1}{2}(\beta-\gamma)=\frac{b-c}{b+c}\operatorname{ctg}\frac{1}{2}\alpha$		$\beta=\frac{1}{2}(\beta+\gamma)+\frac{1}{2}(\beta-\gamma)$ $\gamma=\frac{1}{2}(\beta+\gamma)-\frac{1}{2}(\beta-\gamma)$		$a=b\cdot\frac{\sin\alpha}{\sin\beta}$ $=c\cdot\frac{\sin\alpha}{\sin\gamma}$
			º ' "	b	280,74	b	
		½ α	35º36'23"	c	247,66	sinβ	
		½ (β+γ)	54º23'37"	b-c	+33,08	sinα	
		½ (β-γ)	4º59'47"	b+c	+528,40	sinγ	
		α	71º12'46"			c	
		β				a	
		γ		ctg½ α	1,39646		
		π	180 00 00	tg½ (β-γ)	0,08742	a	

Napomena: ugao π=180º00'00"

4)Računamo uglove β i γ i vrednosti sinusa svih uglova u trouglu:

$$\beta=\frac{1}{2}(\beta+\gamma)+\frac{1}{2}(\beta-\gamma)=54^{o}23'37"+4^{o}59'47"\rightarrow\beta=59^{o}23'24"$$

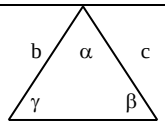
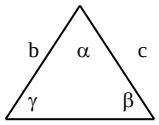
$$\gamma=\frac{1}{2}(\beta+\gamma)-\frac{1}{2}(\beta-\gamma)=54^{o}23'37"-4^{o}59'47"\rightarrow\gamma=49^{o}23'50"$$

Kontrola: α+β+γ= 71º12’46’’+59º23’24’’+49º23’50’’=180º00’00’’ (dozvoljeno odstupanje je ±03’’)

sinα=sin(71º12’46’’)→ sinα=0,94672

sinβ=sin(59º23’24’’)→ sinβ=0,86065

sinγ=sin(49º23’50’’)→ sinγ=0,75924

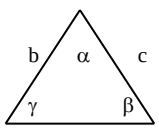
Trigonometrijski obrazac br.14							
Redni broj računanja i skica trougla	Strane i uglovi su uzeti:		$\frac{1}{2}(\beta+\gamma)=\frac{1}{2}\pi-\frac{1}{2}\alpha$ $\operatorname{tg}\frac{1}{2}(\beta-\gamma)=\frac{b-c}{b+c}\operatorname{ctg}\frac{1}{2}\alpha$		$\beta=\frac{1}{2}(\beta+\gamma)+\frac{1}{2}(\beta-\gamma)$ $\gamma=\frac{1}{2}(\beta+\gamma)-\frac{1}{2}(\beta-\gamma)$		$a=b\cdot\frac{\sin\alpha}{\sin\beta}$ $=c\cdot\frac{\sin\alpha}{\sin\gamma}$
			º ' "	b	280,74	b	
		½ α	35º36'23"	c	247,66	sinβ	0,86065
		½ (β+γ)	54º23'37"	b-c	+33,08	sinα	0,94672
		½ (β-γ)	4º59'47"	b+c	+528,40	sinγ	0,75924
		α	71º12'46"			c	
		β	59º23'24"			a	
		γ	49º23'50"	ctg½ α	1,39646		
		π	180 00 00	tg½ (β-γ)	0,08742	a	

5)Računamo dužinu strane a:

$$a=b\cdot\frac{\sin\alpha}{\sin\beta}=280,74\cdot\frac{0,94672}{0,86065}\rightarrow a=308,81m$$

Kontrola :

$$a=c\cdot\frac{\sin\alpha}{\sin\gamma}=247,66\cdot\frac{0,94672}{0,75924}\rightarrow a=308,81m$$

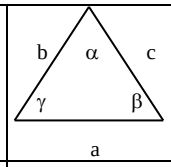
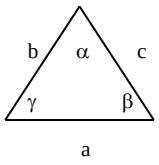
Trigonometrijski obrazac br.14							
Redni broj računanja i skica trougla	Strane i uglovi su uzeti:		$\frac{1}{2}(\beta+\gamma)=\frac{1}{2}\pi-\frac{1}{2}\alpha$ $\operatorname{tg}\frac{1}{2}(\beta-\gamma)=\frac{b-c}{b+c}\operatorname{ctg}\frac{1}{2}\alpha$		$\beta=\frac{1}{2}(\beta+\gamma)+\frac{1}{2}(\beta-\gamma)$ $\gamma=\frac{1}{2}(\beta+\gamma)-\frac{1}{2}(\beta-\gamma)$		$a=b\cdot\frac{\sin\alpha}{\sin\beta}$ $=c\cdot\frac{\sin\alpha}{\sin\gamma}$
			º ' "	b	280,74	b	
		½ α	35º36'23"	c	247,66	sinβ	0,86065
		½ (β+γ)	54º23'37"	b-c	+33,08	sinα	0,94672
		½ (β-γ)	4º59'47"	b+c	+528,40	sinγ	0,75924
		α	71º12'46"			c	
		β	59º23'24"			a	308,81
		γ	49º23'50"	ctg½ α	1,39646		
		π	180 00 00	tg½ (β-γ)	0,08742	a	308,81

Zadatak 2

Rešiti trougao primenom tangensne teoreme, ako su poznati sledeći podaci:

ugao α = 64º28'52", strana b=231,15m i strana c=255,84m

1) Unosimo poznate vrednosti u 14. trigonometrijski obrazac :

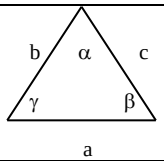
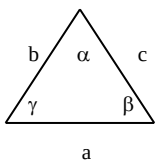
Trigonometrijski obrazac br.14							
Redni broj računanja i skica trougla	Strane i uglovi su uzeti:	 $\frac{1}{2}(\beta+\gamma)=\frac{1}{2}\pi-\frac{1}{2}\alpha$ $\operatorname{tg}\frac{1}{2}(\beta-\gamma)=\frac{b-c}{b+c}\operatorname{ctg}\frac{1}{2}\alpha$			$\beta=\frac{1}{2}(\beta+\gamma)+\frac{1}{2}(\beta-\gamma)$ $\gamma=\frac{1}{2}(\beta+\gamma)-\frac{1}{2}(\beta-\gamma)$		$a=b\cdot\frac{\sin\alpha}{\sin\beta}$ $=c\cdot\frac{\sin\alpha}{\sin\gamma}$
		½ α	º ' "	b	231,15	b	
				c	255,84	sinβ	
		½ (β+γ)		b-c		sinα	
		½ (β-γ)		b+c		sinγ	
		α	64º28'52"			c	
		β				a	
		γ		ctg½ α			
		π	180 00 00	tg½ (β-γ)		a	

2) Računamo razliku i zbir poznatih strana, kao i polovinu ugla α, zatim dobijene vrednosti unosimo u odgovarajuće polje obrasca.

b-c=231,15-255,84= -24,69

b+c=231,15+255,84= +486,99

½ α = $\frac{64^{o}28'52''}{2}$ = 32º14'26"

Trigonometrijski obrazac br.14							
Redni broj računanja i skica trougla	Strane i uglovi su uzeti:	 $\frac{1}{2}(\beta+\gamma)=\frac{1}{2}\pi-\frac{1}{2}\alpha$ $\operatorname{tg}\frac{1}{2}(\beta-\gamma)=\frac{b-c}{b+c}\operatorname{ctg}\frac{1}{2}\alpha$			$\beta=\frac{1}{2}(\beta+\gamma)+\frac{1}{2}(\beta-\gamma)$ $\gamma=\frac{1}{2}(\beta+\gamma)-\frac{1}{2}(\beta-\gamma)$		$a=b\cdot\frac{\sin\alpha}{\sin\beta}$ $=c\cdot\frac{\sin\alpha}{\sin\gamma}$
		½ α	º ' "	b	231,15	b	
			32º14'26"	c	255,84	sinβ	
		½ (β+γ)		b-c	-24,69	sinα	
		½ (β-γ)		b+c	+486,99	sinγ	
		α	64º28'52"			c	
		β				a	
		γ		ctg½ α			
		π	180 00 00	tg½ (β-γ)		a	

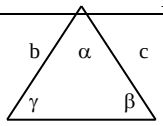
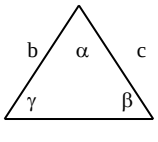
3) Računamo ctg½ α, $\frac{1}{2}(\beta+\gamma)$ i $\frac{1}{2}(\beta-\gamma)$:

ctg½ α= (tg(32º14'26"))$^{-1}$ =0,63065^{-1}= 1,58548

$$\frac{1}{2}(\beta+\gamma)=\frac{1}{2}\pi-\frac{1}{2}\alpha\rightarrow\frac{180^{o}}{2}-32^{o}14'26"\rightarrow\frac{1}{2}(\beta+\gamma)=57^{o}45'34"$$

$$\operatorname{tg}\frac{1}{2}(\beta-\gamma)=\frac{b-c}{b+c}\operatorname{ctg}\frac{1}{2}\alpha\rightarrow\frac{-24,69}{486,99}\cdot 1,58548\rightarrow\operatorname{tg}\frac{1}{2}(\beta-\gamma)=-0,08038$$

$$\frac{1}{2}(\beta-\gamma)=\operatorname{arctg}(-0,08038)=-4^{o}35'45"$$

Trigonometrijski obrazac br.14							
Redni broj računanja i skica trougla	Strane i uglovi su uzeti:		$\frac{1}{2}(\beta+\gamma)=\frac{1}{2}\pi-\frac{1}{2}\alpha$ $\operatorname{tg}\frac{1}{2}(\beta-\gamma)=\frac{b-c}{b+c}\operatorname{ctg}\frac{1}{2}\alpha$		$\beta=\frac{1}{2}(\beta+\gamma)+\frac{1}{2}(\beta-\gamma)$ $\gamma=\frac{1}{2}(\beta+\gamma)-\frac{1}{2}(\beta-\gamma)$		$a=b\cdot\frac{\sin\alpha}{\sin\beta}$ $=c\cdot\frac{\sin\alpha}{\sin\gamma}$
			º ' "	b	231,15	b	
		½ α	32º14'26"	c	255,84	sinβ	
		½ (β+γ)	57º45'34"	b-c	-24,69	sinα	
		½ (β-γ)	-4º35'45"	b+c	+486,99	sinγ	
		α	64º28'52"			c	
		β				a	
		γ		ctg½ α	1,58548		
		π	180 00 00	tg½ (β-γ)	-0,08038	a	

Napomena: ugao π=180º00'00"

4)Računamo uglove β i γ i vrednosti sinusa svih uglova u trouglu:

$$\beta=\frac{1}{2}(\beta+\gamma)+\frac{1}{2}(\beta-\gamma)=57^{o}45'34"+(-4^{o}35'45")\rightarrow\beta=53^{o}09'49"$$

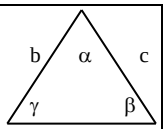
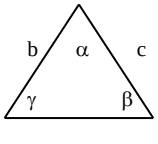
$$\gamma=\frac{1}{2}(\beta+\gamma)-\frac{1}{2}(\beta-\gamma)=57^{o}45'34"-(-4^{o}35'45")\rightarrow\gamma=62^{o}21'19"$$

Kontrola: α+β+γ= 64º28’52’’+53º09’49’’+62º21’19’’=180º00’00’’ (dozvoljeno odstupanje je ±03’’)

sinα=sin(64º28’52’’)→ sinα=0,90244

sinβ=sin(53º09’49’’)→ sinβ=0,80035

sinγ=sin(62º21’19’’)→ sinγ=0,88584

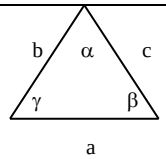
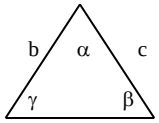
Trigonometrijski obrazac br.14							
Redni broj računanja i skica trougla	Strane i uglovi su uzeti:		$\frac{1}{2}(\beta+\gamma)=\frac{1}{2}\pi-\frac{1}{2}\alpha$ $\operatorname{tg}\frac{1}{2}(\beta-\gamma)=\frac{b-c}{b+c}\operatorname{ctg}\frac{1}{2}\alpha$		$\beta=\frac{1}{2}(\beta+\gamma)+\frac{1}{2}(\beta-\gamma)$ $\gamma=\frac{1}{2}(\beta+\gamma)-\frac{1}{2}(\beta-\gamma)$		$a=b\cdot\frac{\sin\alpha}{\sin\beta}$ $=c\cdot\frac{\sin\alpha}{\sin\gamma}$
			º ' "	b	231,15	b	
		½ α	32º14'26"	c	255,84	sinβ	0,80035
		½ (β+γ)	57º45'34"	b-c	-24,69	sinα	0,90244
		½ (β-γ)	-4º35'45"	b+c	+486,99	sinγ	0,88584
		α	64º28'52"			c	
		β	53º09'49"			a	
		γ	62º21'19"	ctg½ α	1,58548		
		π	180 00 00	tg½ (β-γ)	-0,08038	a	

5)Računamo dužinu strane a:

$$a = b \cdot \frac{\sin\alpha}{\sin\beta} = 231{,}15 \cdot \frac{0{,}90244}{0{,}80035} \rightarrow a = 260{,}63\text{m}$$

Kontrola :

$$a = c \cdot \frac{\sin\alpha}{\sin\gamma} = 255{,}84 \cdot \frac{0{,}90244}{0{,}88584} \rightarrow a = 260{,}63\text{m}$$

Trigonometrijski obrazac br.14							
Redni broj računanja i skica trougla	Strane i uglovi su uzeti:		$\frac{1}{2}(\beta+\gamma)=\frac{1}{2}\pi-\frac{1}{2}\alpha$ $\text{tg}\frac{1}{2}(\beta-\gamma)=\frac{b-c}{b+c}\text{ctg}\frac{1}{2}\alpha$		$\beta=\frac{1}{2}(\beta+\gamma)+\frac{1}{2}(\beta-\gamma)$ $\gamma=\frac{1}{2}(\beta+\gamma)-\frac{1}{2}(\beta-\gamma)$		$a=b\cdot\frac{\sin\alpha}{\sin\beta}$ $=c\cdot\frac{\sin\alpha}{\sin\gamma}$
			o ' "	b	231,15	b	
		½ α	32º14'26"	c	255,84	sinβ	0,80035
		½ (β+γ)	57º45'34"	b-c	-24,69	sinα	0,90244
		½ (β-γ)	-4º35'45"	b+c	+486,99	sinγ	0,88584
		α	64º28'52"			c	
		β	53º09'49"			a	260,63
		γ	62º21'19"	ctg½ α	1,58548		
		π	180 00 00	tg½ (β+γ)	-0,08038	a	260,63

Svaki student je u obavezi da pošalje e-mail sa brojem indeksa, da bi dobio potrebne podatke.
Konsultacije: putem mail-a
(utorkom od 14h do 17h i četvrtkom od 14 do 16h)